FYIT Semester-1
Discrete Maths
Assignment 1
Unit 2
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Q1. Let $B = \{-56, -39, -5, 0, 8, 12, 50, 56\}$. Determine which of the following statements are true or false:

(i) $\forall x \in B$, if x is even then $x > 0$.

(ii) $\exists x \in B$, such that x and $-x$ both are in B.

(iii) $\forall x \in B$, if x is odd then $x < 0$.

(iv) $\forall x \in B$, if x is non-zero even number then it is divisible by 4.

(v) $\forall x \in B$, if the ones digit of x is 6, then tens digit is -5 or 5.

Q2. Negate each of the following statements.

(a) $\exists x \, \forall y \, P(x, y)$

(b) $\forall x \, \forall y \, \neg P(x, y)$

(c) $\exists y \, \exists x \, \forall z \, P(x, y, z)$

(d) $\forall x \, \exists y \, \exists z \, P(x, y, z)$

(e) $\forall x \in A \, \exists y \in B \, (P(x, y))$

Q3. Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of each of the following statements.

(a) Let $P(x) : x + 3 = 10$, $\exists x \in A$, $P(x)$

(b) Let $P(x) : x + 3 < 10$, $\forall x \in A$, $P(x)$

Q4. Convert the following arguments using quantifiers.

(i) All healthy people are vegetarian.
Mona is not healthy.
Conclusion: Mona is not vegetarian.

(ii) All healthy people do exercise every day.
Ruby does exercise every day.
Conclusion: Ruby is a healthy person.

(iii) No good car is cheap.
Ford is not cheap.
Conclusion: Ford is a good car.

Q5. Prove directly that:

(i) Prove directly that the difference of an odd and an even integer is odd.

(ii) Prove that the difference of any two rational numbers is a rational number.

(iii) If $r, s \in Z$ such that r is even and s is odd then $r^2 + 3s$ is odd.

Q6. Prove by Contraposition or contradiction.

(i) For all integers z, if z^2 is even then z is even.

(ii) For all $x, y \in Z$, if $x + y$ is even then either both x and y are even or both are odd

(iii) For all $x, y \in Z$, if $x + y < 50$ then either $x < 25$ or $y < 25$

(iv) $\sqrt{6} - 7\sqrt{2}$ is irrational.

Q7. Proof by cases:

(i) Prove: $\forall x \in Z, x^4 = 8n$ or $x^4 = 8n + 1$

(iii) $\forall x, y \in Z, y \neq x$ and $y - x$ both are even, both odd and one even and one odd

(iv) $\lfloor \frac{x}{2} \rfloor + \lceil \frac{x}{2} \rceil = x \forall x \in Z$

Q8. Use quotient-reminder theorem to prove that the product of any two Consecutive integer has the form 3p or 3p+2 for some k $\in Z. Taken = 3$.

Q9. Prove that $\forall x \in Z$**, (a) if** $x \bmod 5 = 3$ **then** $x^2 \bmod 5 = 4$**; (b) if** $x \bmod 7 = 6$ **then** $(5x - 3) \bmod 7 = 6$

Q10. Prove that $m(m^2 - 1)(m + 2)$ **is divisible by 4 for any** $m \in Z$**.**