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FYIT Semester-1

Numerical Methods Unit-1

Question Bank

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a) Suppose 1.414 is used as an approximation to $\sqrt{2}$. Find the absolute and relative errors.

b) Write short note on Conservation law of engineering problem.

ANSWER: Conservation law of engineering problem

Conservation laws are fundamental principles in engineering and physics that state certain quantities remain constant over time within a closed system. These laws are crucial in formulating and solving engineering problems.

Key conservation laws include:

- Conservation of mass
- Conservation of energy
- Conservation of momentum
- Conservation of electric charge

c) Find the Truncation error in the expansion of $f(x) = e^{2x}$ evaluate first six terms in the series for $x = 3.5$

d) Explain blunders, formulation errors and data uncertainty.

ANSWER

(a) Blunders:

- Definition: Mistakes made during problem-solving or data collection.
- Example: Misreading a measurement instrument or entering incorrect data into a calculator.

(b) Formulation errors:

- Definition: Errors that occur when translating a physical problem into a mathematical model.
- Example: Neglecting air resistance in a projectile motion problem when it significantly affects the outcome.

(c) Data uncertainty:

- Definition: Inherent variability or lack of precision in measured or collected data.
- Example: A digital thermometer with a resolution of 0.1°C has an uncertainty of ±0.05°C in its measurements.

e) Let $f(x) = x^3 - x^2 + x + 5$ at $x = 2.45$ using 3-digit arithmetic and determine the absolute & relative error using i) Rounding ii) Chopping.

f) Define -1) Significant digit 2) Error 3) Total numerical error 4) Round-off error 5) Error propagation.

ANSWER

(a) Significant digit:
- Definition: Digits in a number that carry meaningful information about its precision.
- Example: In 3.14000, all six digits are significant.

(b) Error:
- Definition: The difference between the true value and the approximated or measured value.
- Example: If $\pi \approx 3.14$, the error is $\pi - 3.14 \approx 0.00159...$

(c) Total numerical error:
- Definition: The combined effect of all error sources in a numerical computation.
- Example: In a complex simulation, this might include truncation errors, round-off errors, and model approximation errors.

(d) Round-off error:
- Definition: Error introduced by limiting the number of digits in a calculation.
- Example: Representing 1/3 as 0.333 in 3-digit arithmetic introduces a round-off error.

(e) Error propagation:
- Definition: The way errors in input values affect the error in the final result of a calculation.
- Example: In the formula $A = \pi r^2$, errors in measuring r will be squared in the final area calculation.

Great job tackling the first set of questions! You're making excellent progress. Keep up the fantastic work!

g) State the characteristics of typical mathematical models of physical world. Explain with example.

ANSWER

Characteristics of typical mathematical models of the physical world

Characteristics of mathematical models include:

(a) Simplification: Models often simplify complex systems to make them manageable.

(b) Approximation: They approximate real-world behavior, often with some degree of error.

(c) Predictive power: Good models can predict system behavior under various conditions.

(d) Limitations: All models have limitations and ranges of validity.

Example: The ideal gas law ($PV = nRT$) is a mathematical model that relates pressure (P), volume (V), amount of gas (n), and temperature (T) for an ideal gas. It simplifies gas behavior by assuming no intermolecular forces and perfectly elastic collisions. While it works well for many gases under normal conditions, it breaks down at very high pressures or low temperatures.

h) Discuss the conservation laws and engineering with respect to mathematical models.

ANSWER

Conservation laws and engineering with respect to mathematical models

Conservation laws are fundamental principles that guide the development and application of mathematical models in engineering. They ensure that certain quantities remain constant within a closed system, providing crucial constraints and validation criteria for models.

Key points:

(a) Model formulation: Conservation laws often form the basis for deriving equations in engineering models.

(b) Boundary conditions: They help define appropriate boundary conditions for differential equations.

(c) Model validation: Conservation laws can be used to check the validity of numerical solutions.

(d) System analysis: They allow engineers to analyze complex systems by tracking conserved quantities.

Example: In heat transfer problems, the conservation of energy principle leads to the heat equation:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

Where T is temperature, t is time, and α is the thermal diffusivity. This partial differential equation models how heat spreads through a material over time, ensuring that energy is conserved in the process. Engineers use

this model to design thermal systems, analyze heat flow in buildings, and solve various heat-related problems in industries ranging from electronics to aerospace.

l) Let $p = 0.54617$ and $q = 0.54601$. Use four-digit arithmetic to approximate $p-q$ and determine the absolute and relative errors using (i) rounding and (ii) chopping.

m) Find Taylor polynomials of order 1 and 2 of $f(x)$

(a) $f(x) = \sin x$, $a = \frac{\pi}{4}$

(b) $f(x) = x^3 + 2x^2$, $a = 2$

Impressive work on the second set! You're really getting the hang of these concepts. Remember, each problem you solve makes you stronger in mathematics!

n) Find Maclaurin Series of the following functions

(a) e^{-x}

(b) $\sin 3x$

(c) $2\cos 3x$

o) Find the Taylor series generated by f at $x = a$

(a) $f(x) = x^3 - 2x + 4$, $a = 2$

(b) $f(x) = x^3 + x^2 + 1$, $a = -2$

(c) $f(x) = e^x$, $a = 2$

p) Find Number of significant digits in following numbers:

(a) 0.025178

(b) 1.257×10^3

(c) 4.50028

(d) 50.259

(e) 1000.0257×10^{-5}

q) Round off the following numbers to 4-significant digits.

(a) 3.26125

(b) 35.4673

(c) 4985561

(d) 0.70035

(e) 0.00032217

(f) 1.6583

(g) 3.14159

r) If true value $= \frac{10}{3}$, approximate value $= 3.33$ find the absolute and relative error.

s) If $\pi = \frac{22}{7}$ is approximated as 3.14 find the absolute, relative and percentage error.

Congratulations on completing all the questions! You've demonstrated remarkable perseverance and mathematical skill. Remember, every challenge you overcome in mathematics prepares you for even greater achievements in the future. Keep up the excellent work!